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THE PRICE ELASTICITY OF DERIVED DEMAND FOR URBAN RESIDENTIAL LAND

C. F. Sirmans, Assistant Professor of Finance

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University of Illinois at Urbana-Champaign

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Summary:

The purpose of this paper is to provide some preliminary estimates of the price elasticity of demand for residential sites for 52 urban areas in the United States. Estimates are also made for three time periods, 1967, 1971 and 1975 in order to investigate changes in the price elasticity over time. Using Marshall's well known four rules for the price elasticity of demand for an input factor, the results indicate a substantial variation in the price elasticity of demand for land both across cities and over time. An inelastic demand was found for all cities in the sample and for all three time periods.

THE PRICE ELASTICITY OF DERIVED DEMAND FOR URBAN RESIDENTIAL LAND

> "The demand for raw materials and other means of production is indirect and derived from the direct demand for those directly serviceable products which they help to produce."
>
> Alfred Marshall

I. Introduction

Numerous studies have investigated the determinants of urban land values.[1] These studies may be divided into those analyzing intraurban and interurban variations.[2] Marginal productivity theory, which forms the basis of factor price determination, has received recent application to the theory of land rent and land use.[3] If land is viewed as an input into the production of goods and services, for example housing services, the demand for land as a factor is a derived demand.[4] Neoclassical theory of factor price determination tells us that firms and industries will employ the level of factors only insofar as their employment contributes to the goods and services valued by consumers. Another question of importance, which has received little attention in the literature, concerns the determinants of not the level of factor demand but of its elasticity, i.e., its responsiveness to changes in factor prices.[5] Knowledge of this price elasticity could provide important insights into understanding the housing problems of urban areas.

The purpose of this study is to provide some estimates of the price elasticity of demand for residential sites for 52 urban areas in the United States. Estimates will also be made for three time periods, 1967, 1971 and 1975 in an attempt to understand how the price elasticity has changed over time in these areas. The paper can be outlined as follows. Section II outlines the methodology for estimating the price elasticity of an input factor, in this case land; Section III provides some estimates for 52 urban areas in 1967, 1971 and 1975; finally, summary and conclusions of this study are in the final section.

II. Methodology

The theory of derived demand links distribution theory and the supply conditions of productive factors.[6] The elasticity of derived demand for land as an input factor has received little attention in land economics literature. Since the demand for residential land is created by the need to produce a product for sale, in this case housing services, the demand for land is a derived demand. The forces influencing the elasticity of demand for a productive factor, such as land, are: (i) the elasticity of demand for the final product, (ii) the elasticity of substitution in production, (iii) the elasticity of supply of other input factors, and (iv) the relative importance of the factor.

In a model of housing production, the price elasticity of derived demand may be written formally using the following

definitions. Assume a production function for housing which may be written,

$$Q = f(L, K) \tag{1}$$

where Q is output of housing services, L is land input, and K is capital (structures) input. The production function is assumed to be a continuous, twice differentiable homogeneous function of the first degree. The elasticity of demand for housing services is

$$\eta = -\frac{dQ}{dp}\cdot\frac{p}{Q} \tag{2}$$

where p is the price of housing services. The elasticity of derived demand for land is

$$\lambda = -\frac{dL}{dr}\cdot\frac{r}{L} \tag{3}$$

where r is the unit price of the land input. The elasticity of supply of capital input is

$$e_k = \frac{dK}{dn}\cdot\frac{n}{K} \tag{4}$$

where n is the unit price of the capital input. The relative shares of the land and capital factor inputs are

$$\alpha_L = \frac{rL}{pQ}\text{ , and} \tag{5}$$

$$\alpha_K = \frac{nK}{pQ} \tag{6}$$

where $\alpha_L + \alpha_K = 1$. The elasticity of substitution between land and capital is

$$\sigma = [d(K/L)/(K/L)]/[d(r/n)/(r/n)]. \tag{7}$$

Under the assumptions of competition in both factor and product markets and constant returns to scale in the production function given in equation (1), the elasticity of derived demand can be expressed as

$$\lambda = \frac{\eta\sigma + e_K(\alpha_L\eta + \alpha_K\sigma)}{\alpha_L\sigma + \alpha_K\eta + e_K} \quad . \tag{8}$$

Equation (8) is exactly the same as Hicks' formulation and requires that the price elasticity of demand for housing services be treated as a positive number for calculation purposes.[7]

Marshall's well-known four rules of derived demand, under the special case of fixed factor proportions as applied to land, are:[8]

(1) The demand for land is likely to be more elastic, the more elastic is the demand for the output it contributes to produce, in this case housing services.

(2) The demand for land is likely to be more elastic, the more readily substitutes for land in the production process can be obtained.

(3) The demand for land is likely to be more elastic, the more elastic is the supply of its cooperate agents of production.

(4) The demand for land is likely to be less elastic, the less important is

the part played by the cost of land in the total cost of the production of housing.

Hicks (1932) confirmed these rules for the general case where the elasticity of substitution is not necessarily equal to zero, with the exception of the fourth rule.[9] Only if the elasticity of demand for the final product is greater than the elasticity of substitution between factors of production is the fourth rule true. If the demand for the final product, housing services, is fairly elastic, while substitution is difficult, the conditions will hold for the fourth rule. This "importance of being unimportant" condition is true only when the consumer can substitute more easily than the housing producer.

The next section uses equation (8) to estimate the price elasticity of land for 52 housing areas in 1967, 1971 and 1975.

III. Estimates

In order to estimate the price elasticity of derived demand for land, λ, estimates are needed for the price elasticity of demand for housing services, η, the elasticity of substitution, σ, the elasticity of supply of capital input, e_K, and the relative factor share of land, α_L. Since any or all of these may vary across urban areas, this section will first discuss the estimates of these parameters to be used in this paper. Following this discussion, the price elasticity of land will be presented and discussed.

Numerous studies have examined the price elasticity of demand for housing, η. Kau and Lee (1976) have recently provided a comprehensive study of elasticity of demand for housing in 50 SMSA's in 1970. While Kau and Lee (1976) were unable to provide direct estimates of the price elasticity of demand for housing services, they were able to shed some light on the magnitude of the elasticity.[10] The Kau and Lee study included 28 of the 52 housing areas included in the sample of the present study. Of the 50 cities included in the Kau and Lee study, approximately 50 percent indicated a price elasticity of demand for housing services which was significantly less than unity in absolute value. For the other 50 percent the hypothesis that the price elasticity was equal to minus one could not be rejected. One of the interesting findings of their study was that no city exhibited a significant elastic demand for housing. These results tend to imply that the price elasticity of demand for housing services is between zero and minus one, with it being exactly minus one in half of their sample.

Other studies which have examined the demand price elasticity for housing services include Muth (1972), Reid (1962) and Lee (1964). These estimates for all types of housing services indicate a price elasticity of unity or slightly higher in absolute value. While the literature lacks a comprehensive study of interurban variation in the price elasticity, the above estimates, particularly the Kau and Lee study, provide a strong case for a demand elasticity

of minus one.[11] This estimate is used to calculate results of the price elasticity of demand for residential sites in Table 1.

Using a variable elasticity of substitution (VES) production function, Sirmans and Redman (1977) have recently provided estimates of the elasticity of substitution, σ, between land and structures for the 52 FHA housing areas in 1967, 1971 and 1975.[12] This VES function allows the elasticity to vary with the capital-land ratio. The data included in the Sirmans and Redman study was new single-family housing financed through the FHA program. The estimates made by Sirmans and Redman of the elasticity are included in Table 1. These estimates show a wide variation in σ across cities and over the three time periods. The elasticity of substitution between land and structures declined in 48 of the 52 areas between 1967 and 1975, indicating that it was more difficult to substitute capital for land in most housing areas.

The weakest empirical evidence is available for the elasticity of supply of other inputs, e_K, in the production of housing. The literature which does exist, Muth (1972) and Clawson (1971), makes a strong case that there probably exists a highly elastic supply.[13] These arguments are based on the relative ease with which builders move in and out of the construction industry. No literature exists on the interurban variation in the elasticity of supply. The estimate used in this paper is that by Muth (1969, p. 317)

Table 1

PRICE ELASTICITY OF DEMAND FOR RESIDENTIAL SITES IN 52 HOUSING AREAS: 1967, 1971 and 1975

	Elasticity of Substitution $(\sigma)^a$			Land Share $(\alpha_L)^b$			Price Elasticity of Demand $(\lambda)^c$		
City	1967	1971	1975	1967	1971	1975	1967	1971	1975
Albuquerque	.3527	.3839	.3273	.1582	.1507	.1357	.4515	.4736	.4152
Birmingham	.6684	.7436	.6520	.1319	.1342	.0958	.7113	.7775	.6846
Boise City	.4681	.5130	.6055	.1111	.1322	.1680	.5254	.5756	.6704
Charlotte	.6755	.6656	.6140	.1750	.1556	.1663	.7313	.7167	.6768
Chattanooga	.7213	.7352	.6933	.1227	.1188	.1104	.7549	.7661	.7265
Chicago	.4822	.3758	.3189	.1822	.1614	.1518	.5740	.4732	.4185
Cincinnati	.6211	.5457	.4781	.2098	.1951	.1793	.6990	.6322	.5691
Columbus, OH	.5936	.5045	.4056	.2078	.1852	.1771	.6763	.5939	.5076
Dallas	.4973	.4212	.2575	.1498	.1924	.1601	.5705	.5292	.3717
Dayton	.6190	.5246	.5126	.2338	.2029	.1885	.7064	.6187	.6021
Denver	.4712	.5442	.4344	.1684	.1690	.1363	.5577	.6193	.5091
Detroit	.5622	.5122	.4916	.1884	.2023	.1987	.6428	.6084	.5900
El Paso	.4423	.3779	.2686	.1514	.1463	.1365	.5242	.4658	.3644
Ft. Lauderdale	.3774	.3449	.2970	.1788	.1525	.2481	.4851	.4413	.4655
Fresno	.2617	.3041	.2249	.1893	.1969	.1881	.3961	.4362	.3649
Greensboro, NC	.6684	.6206	.5522	.1489	.1564	.1703	.7169	.6787	.6266
Houston	.3770	.2851	.2759	.1563	.1521	.1448	.4711	.3896	.3766
Jacksonville	.4698	.5122	.3983	.1386	.1250	.1631	.5411	.5715	.4933
Kansas City	.5982	.5900	.5111	.1697	.1513	.1522	.6649	.6506	.5835
Knoxville	.7397	.7320	.6658	.1393	.1369	.1084	.7754	.7681	.7013
Las Vegas	.3389	.3512	.2544	.1834	.2284	.1818	.4560	.4947	.3847
Little Rock	.5517	.5875	.5445	.1716	.1689	.1354	.6268	.6556	.6046
Louisville	.5940	.5780	.5161	.1948	.1934	.1775	.6714	.6578	.5998
Lubbock	.3696	.3973	.2637	.1315	.1167	.0854	.4496	.4652	.3239
Memphis	.5792	.5203	.4774	.1597	.1573	.1518	.6449	.5938	.5545
Miami	.5319	.3832	.3496	.2046	.1622	.2499	.6254	.4799	.5071
Minneapolis	.6730	.6125	.6185	.1828	.1849	.1742	.7317	.6827	.6836
Montgomery	.6056	.6464	.4912	.1678	.1675	.1451	.6704	.7045	.5630
Nashville	.7411	.6975	.6946	.1794	.1757	.1732	.7869	.7498	.7466
Newport News	.5668	.5977	.4520	.2150	.2271	.2120	.6579	.6872	.5650
Norfolk	.5023	.4057	.2253	.2043	.1895	.2017	.6014	.5149	.3754
Oklahoma City	.5546	.5246	.3665	.1523	.1435	.1249	.6208	.5910	.4428
Omaha	.5270	.5886	.4490	.1600	.1402	.1625	.6007	.6449	.5359

Table 1 - Continued

City	Elasticity of Substitution (σ)[a]			Land Share (α_L)[b]			Price Elasticity of Demand (λ)[c]		
	1967	1971	1975	1967	1971	1975	1967	1971	1975
Orlando	.5298	.5077	.4359	.1744	.1569	.2176	.6098	.5829	.5552
Pensacola	.6420	.5857	.5725	.1488	.1489	.1045	.6942	.6460	.6161
Philadelphia	.3785	.4444	.2475	.1795	.1876	.1877	.4864	.5456	.3833
Phoenix	.2691	.3132	.5507	.1905	.1875	.1921	.4031	.4374	.6350
Pittsburg	.6850	.6617	.6332	.2038	.1745	.1930	.7481	.7196	.7026
Portland	.6159	.5766	.4682	.1786	.1896	.1890	.6831	.6551	.5659
Raleigh	.7093	.6860	.6358	.1669	.1598	.1770	.7570	.7353	.6990
Richmond	.6120	.6220	.3930	.1798	.1851	.1614	.6803	.6906	.4878
Sacramento	.3453	.4018	.2560	.2333	.2005	.1656	.4932	.5181	.3743
St. Louis	.5323	.5959	.5480	.1917	.1981	.1486	.6198	.6743	.6135
Salt Lake City	.6226	.5938	.4989	.1719	.1646	.1773	.6860	.6592	.5854
San Diego	.3343	.3452	.2847	.2613	.2376	.2574	.5028	.4958	.4626
Seattle	.5873	.5397	.4524	.1847	.2055	.1679	.6619	.6321	.5417
Shreveport	.5834	.5239	.4904	.1559	.1549	.1538	.6469	.5957	.5666
Spokane	.5598	.6772	.6420	.1034	.1100	.1099	.6041	.7120	.6805
Stockton	.3097	.3037	.2326	.2085	.2117	.1684	.4486	.4460	.3566
Tacoma	.6141	.5302	.5027	.1519	.1636	.1533	.6715	.6051	.5769
Tampa	.5114	.5108	.3358	.1688	.1529	.1530	.5917	.5836	.4338
Tulsa	.5654	.5305	.3874	.1693	.1662	.1457	.6373	.6066	.4737

[a] See Sirmans and Redman (1977) for a discussion of these estimates.

[b] For FHA new housing.

[c] Calculated using equation (8), σ, α_L and assuming $\eta = -1.0$ and $e_K = 15$.

of approximately 15. The elasticity of supply of other inputs is assumed to be constant across housing areas.

The final factor necessary for estimating the price elasticity of derived demand for land is the relative share of sites, α_L, in total costs. The relative shares were taken from the FHA data and are shown in Table 1. These numbers indicate that α_L had decreased for 37 of the 52 areas between 1967 and 1975.

The price elasticity of demand for residential sites, λ, for single family housing for the 52 housing areas in 1967, 1971 and 1975 was estimated using equation (8) and the estimates of η, σ, α_L and e_K.[14] These estimates in absolute terms are shown in Table 1. These estimates indicate that in no city was there an elastic price elasticity of demand for residential land. The price elasticity declined in 49 of the 52 cities between 1967 and 1975 indicating an even greater inelastic demand in the later time period.

Using the mean estimates of σ, α_L, $\eta = -1.0$ and $e_K = 15$, the price elasticity of the entire sample was -.626, -.599 and -.538 for 1967, 1971 and 1975, respectively.[15] The estimates of the price elasticity in Table 1 indicate a substantial range both across cities and over time. In 1967, the range in λ was from -.40 to -.78 in Fresno and Knoxville. In 1975, the range was from -.32 to -.75 in Lubbock and Nashville.

IV. Summary and Conclusions

The purpose of this paper has been to examine interurban variations over time in the price elasticity of derived demand for urban residential land. The theory of derived demand for an input factor, as developed in Marshall's well known four rules, indicates that the price elasticity depends on the price elasticity of the output, the elasticity of substitution between input factors, the elasticity of supply of other input factors and the relative importance of the input factor. Little empirical research has investigated the responsiveness of the demand for land with respect to its price.

Using interurban estimates of the elasticity of substitution and the relative share of land and assuming that the elasticity of demand for housing services and the elasticity of supply of capital inputs are constant across housing areas, estimates of the demand price elasticity for land were made for 52 FHA housing market areas in 1967, 1971 and 1975. These estimates indicate an inelastic demand for residential sites for all housing areas and for all three time periods. The estimates indicated that the demand for land was less elastic in the later time period.

While the estimates should be interpreted as preliminary, they are the first to investigate this neglected area of urban land economics. More refined estimates can be developed when research examines the interurban variations

in the price elasticity of demand for housing services and the elasticity of supply for other inputs in housing production.

Footnotes

[1] See Rubenfeld (1972) for a discussion of the literature.

[2] For examples of the former, see Mills (1969), Kau and Sirmans (1978) and Downing (1970). For examples of the latter, see Muth (1971) and Witte (1975).

[3] See Mills (1972), chapter 3.

[4] Muth (1971) has recently developed a model of demand for land assuming a constant elasticity of substitution production function in housing production. Witte (1975) has provided some empirical estimates of interurban demand for land. Kau and Lee (1976a) have developed a model of urban land-use using the CES function for housing production with land and capital inputs.

[5] Witte (1977) has recently estimated the price elasticity for aggregate U.S. data at -.7. Another area which has been neglected is possible intraurban variations in the price elasticity of land.

[6] For a good discussion of the theory of derived demand, see Stigler (1965). Some recent literature is Bronfenbrenner (1961), Muth (1962), Ferguson (1966), Mundlak (1968), Andrieu (1974) and Sato and Koizumi (1971). Early literature includes Hicks (1932), Marshall (1920) and Pigou (1932).

[7] See Sato and Koizumi (1971) for the derivation of this equation. The Hicks (1963, p. 374) formulation is

$$\lambda = \frac{e_K\alpha_L[\eta-\sigma] + \sigma[\eta+e_K]}{[\eta+e_K] - \alpha_L[\eta-\sigma]}$$

[8] See Marshall (1920), chapter 6, Book V.

[9] Andrieu (1974) has recently generalized equation (8) in the case of non-constant returns to scale in production. Andrieu's generalization can be written

$$\lambda = \frac{e_K\alpha_L[\eta-\sigma E] + \sigma[\eta+e_K E]}{[\eta+e_K E] - \alpha_L[\eta-\sigma E]}$$

where $E = \eta(1-\rho) + \rho$. ρ is the degree of homogeneity of the industry's production function. Obviously, if ρ is equal to one, this equation reduces to the Hicks' formulation in footnote 7. Sato and Koizumi (1971) have introduced different

degrees of substitutability and complimentarity among factors of production to the theory of derived demand.

[10] Kau and Lee (1976b) needed an estimate of α_L which was not available in their sample.

[11] Witte (1977) has argued that $\eta = -1.5$.

[12] Other literature on the elasticity of substitution includes Muth (1971) and Koenker (1972) and Sirmans, Kau and Lee (1978). These studies however do not estimate the interurban variation in the elasticity of substitution.

[13] See also de Leeuw and Ekanem (1971), Greison (1973) and Smith (1976). Smith has investigated the long-run supply elasticities of housing density and quality.

[14] To examine the sensitivity of the assumption of a constant η and e_K, some alternative estimates of λ were made using η equal to -.7, -1.5 and -2.0 and e_K equal to 10 and 20. As rules 1 and 3 state, the price elasticity increased as η and e_K were increased but in all cases the price elasticity was inelastic. These estimates are available in an appendix.

[15] Sirmans and Redman (1977) estimated the mean elasticity of substitution at .55, .52 and .45 for 1967, 1971 and 1975.

References

Andrieu, Michel. 1974. "Derived Demand, Returns to Scale and Stability." Review of Economic Studies 41 (3): 405-417.

Bronfenbrenner, Martin. 1961. "Notes on the Elasticity of Derived Demand." Oxford Economic Papers 13 (3): 254-61.

Clawson, Marion. 1971. Suburban Land Conversion in the United States: An Economic and Governmental Process. Baltimore: Johns Hopkins University Press.

de Leeuw, Frank, and Ekanem, Nkanta F. 1971. "The Supply of Rental Housing." American Economic Review 61 (5): 806-17.

Downing, Paul B. 1970. "Estimating Land Values by Multivariate Analysis." In The Assessment of Land Value, ed. D. M. Holland. Madison: University of Wisconsin Press.

Ferguson, C. E. 1966. "Production, Prices, and the Theory of Jointly Derived Input Demand Functions." Economica 33 (4): 60-75.

Gottlieb, Manuel. 1965. "Influences on Value in Urban Land Market, U.S.A. 1956-61." Journal of Regional Science 6 (1): 1-16.

Grieson, Ronald E. 1973. "The Supply of Rental Housing: Comment." American Economic Review 63 (3): 433-36.

Hicks, J. R. 1932. Theory of Wages. London: Macmillan, (2nd Edition, 1963).

__________. 1961. "Marshall's Third Rule: A Further Comment." Oxford Economic Papers 13 (3): 355-60.

Kau, J. B., and Lee, C. F. 1976a. "Capital-Land Substitution and Urban Land Use." Journal of Regional Science 16 (1): 83-92.

__________. 1976b. "Functional Form, Density Gradient and Price Elasticity of Demand for Housing." Urban Studies 13 (3): 193-98.

Kau, J. B., and Sirmans, C. F. 1978. "Urban Land Value Function and the Price Elasticity of Demand for Housing." Journal of Urban Economics, forthcoming.

Koenker, R. 1972. "An Empirical Note on the Elasticity of Substitution Between Land and Capital in a Monocentric Housing Market." *Journal of Regional Science* 12 (1): 299-305.

Lee, Tong H. 1964. "The Stock Demand Elasticities for Non-Farm Housing." *Review of Economics and Statistics* 46 (1): 82-89.

Marshall, Alfred. 1920. *Principles of Economics*. London: Macmillan.

Mills, E. S. 1972. *Urban Economics*. Glenview, Illinois: Scott-Foresman.

Mundlak, Y. 1968. "Elasticities of Substitution and the Theory of Derived Demand." *Review of Economic Studies* 35 (3): 80-95.

Muth, R. F. 1964. "The Derived Demand Curve for a Production Factor and the Industry Supply Curve." *Oxford Economic Papers* 16 (2): 221-34.

__________. 1969. *Cities and Housing: The Spatial Pattern of Urban Residential Land Use*. Chicago: University of Chicago Press.

__________. 1971. "The Derived Demand for Urban Residential Land." *Urban Studies* 8 (3): 243-54.

__________. 1972. "The Demand for Non-Farm Housing." In *Readings in Urban Economics*, eds. Matthew Edel and Jerome Rothenberg. New York: Macmillan.

Pigou, A. C. 1932. *Economics of Welfare.* London: Macmillan.

Reid, Margaret G. 1962. *Housing and Income*. Chicago: University of Chicago Press.

Rubenfeld, D. 1972. "Urban Land Prices: Empirical and Theoretical Essays." Working Paper No. 13, M.I.T. Joint Center for Urban Studies. Cambridge, Mass.

Sato, Ryuzo and Koizumi, Tetsunori. 1970. "Substitutability, Complementanity and the Theory of Derived Demand." *Review of Economic Studies* 37 (3): 107-18.

Sirmans, C. F., Kau, J. B., and Lee, C. F. 1978. "The Elasticity of Substitution in Urban Housing Production: A VES Approach." *Journal of Urban Economics*, forthcoming.

Sirmans, C. F., and Redman, A. L. 1977. "Capital-Land Substitution: Some Inter-Urban Estimates." Mimeo, Dept. of Finance, University of Illinois, Urbana.

Smith, B. A. 1976. "The Supply of Urban Housing." Quarterly Journal of Economics 65 (2): 385-405.

Stigler, G. J. 1966. The Theory of Price. New York: Macmillan.

U. S. Department of Housing and Urban Development. FHA Homes, 1967, 1971 and 1975. Washington, D. C. Federal Housing Administration.

Witte, Ann D. 1975. "The Determination of Inter-Urban Residential Site Prices Differences: A Derived Demand Model with Empirical Testing." Journal of Regional Science 15 (3): 351-64.

__________. 1977. "An Examination of Various Elasticities for Residential Sites." Land Economics 52 (4): 401-409.

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